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INNER PRODUCT PERFORMANCE CRITERIA FOR EVALUATING COMBAT MODELS

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FEBRUARY 1991

U. S. ARMY MATERIEL SYSTEMS ANALYSIS ACTIVITY
ABERDEEN PROVING GROUND, MARYLAND

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An inner product performance criteria is proposed which has a simple gemoetric meaning and is easy to calculate for evaluating how well the approximate solutions currently used in an Army model agrees with the exact solution associated with the linear homogeneous Lanchester equation of combat.

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INNER PRODUCT PERFORMANCE CRITERIA FOR EVALUATING COMBAT MODELS

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U.S. ARMY MATERIEL SYSTEMS ANALYSIS ACTIVITY
ABERDEEN PROVING GROUND, MARYLAND 21005-5071

The U.S. Army is actively engaged in developing combat models using approximate solutions to the linear homogeneous Lanchester equations. How well these approximate solutions deviate from the true or exact solution is an important issue within the Army if we are going to have any confidence in these approximate solutions. An inner product performance criteria is proposed which has a simple geometric meaning and is easy to calculate for evaluating how well the approximate solution agrees with the exact solution associated with the linear homogeneous Lanchester equation of combat.

It is shown that an inner product performance criteria is a useful measure to evaluate the performance of current combat models against the exact solutions for piecewise linear homogeneous Lanchester equations for all homogeneous or heterogeneous battles.

The Lanchester homogeneous equations in vector form may be given by Eq (1)

$$\dot{x}(t) = A(t) \quad x(t) \qquad\qquad (t_o \leq t \leq t_f) \quad (1)$$

where x is an n x 1 vector such that

$$x(t) \;=\; (R_1 \quad R_2 \ldots R_p, B_1, B_2, \ldots B_q)$$

R_i is the i^{th} Red weapon system strength at time t.

B_j is the j^{th} Blue weapon system strength at time t.

and

$p + q = n$

R_i and B_j both being nonnegative,

with an initial condition $x(t_0) = x_0$, A(t) is the time varying n by n attrition coefficient matrix. Assuming A(t) is piecewise constant for $t_i \leq t \leq t_{i+1}$ for i = 0, 1 ..., n where $t_{n+1} = t_f$, then Eq (1) has an exact solution given by

$$x(t_{i+1}) = e^{(t_{i+1} - t_i)A} x(t_i) \qquad (2)$$

$$for \quad i = 0, \; 1, \; \ldots, \; n$$

We now define the adjoint differential equation of Equation (1) given by

$$\dot{y} = -A^T(t)y \qquad (3)$$

with a value of y(t) at t = t_f given by y (t_f).

We now claim that the inner product $y^T(t)$ x (t) is a constant. This can be shown as follows:

$$\frac{d}{dt} y^T(t)x(t) = \dot{y}^T x(t) + y^T(t)\dot{x}(t)$$

$$= (-A^T y)^T \; x(t) + y^T(t)\dot{x}(t)$$

$$= y^T(-Ax) + y^T(t)\dot{x}(t)$$

$$= -y^T(t)\dot{x}(t) + y^T(t)\dot{x}(t)$$

$$= 0 \qquad QED.$$

where we have used Eq (1) and Eq (3).

Thus

$$y^T(t)x(t) \; = \; constant \qquad (4)$$

or more specifically at the final time, t_f, of the battle

$$y^T(t_f)x(t_f) = y^T(0)x(0) \qquad (5)$$

In fact if we select $y(t_f)$ to be orthogonal to $x\ (t_f)$ we have

$$y^T(t_f)x(t_f) = 0 \qquad (6)$$

and the inner product $y^T(t)\ x\ (t)$ will remain zero for all t. Hence all solutions of the adjoint equation, $y(t)$ in Eq (3), are orthogonal to the surface containing the solution of the Lanchester equation in Eq (1), even for time varying attrition matrix and all battles. The geometric meaning is simply that the projection of $x(t)$ on the adjoint solution $y(t)$ multiplied by the magnitude of $y(t)$ is zero. Figure 1 provides the geometric relationship and the position of a generic Army Model solution, x_{AM}, in relationship to x and y.

Within the Army, the model given by x_{AM}, for Army Model, uses the Lanchester equation to describe many combat situations. The finite difference equation for Eq (1) is

$$x_{AM}(t + \Delta t) = (I + A\Delta t)x_{AM}(t)$$

which with $\Delta t = 1$

$$x_{AM}(t + 1) = (I + A) \quad x_{AM} \qquad (7)$$

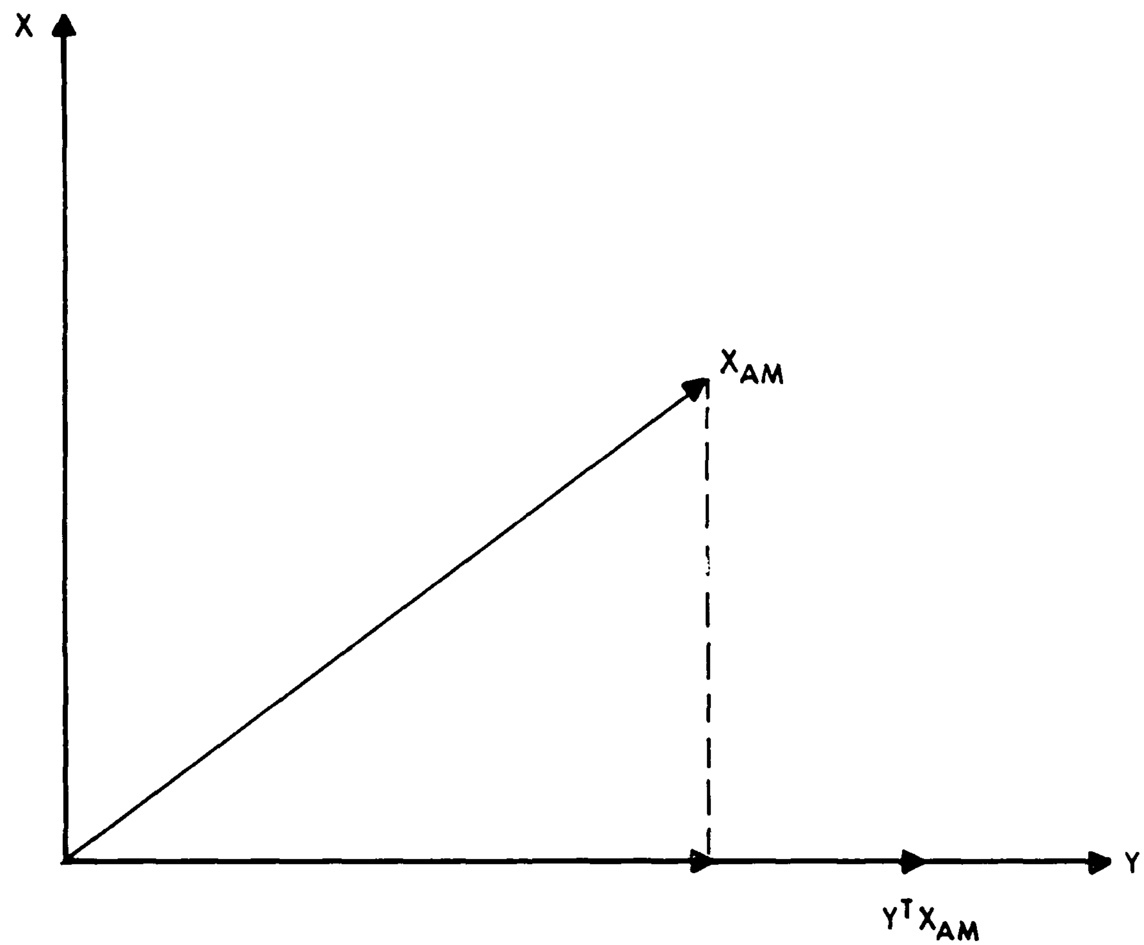
If X_{AM} is to be a close approximation to x, then the inner product of x_{AM} with y of the adjoint equation, Eq (3), is the projection of x_{AM} along y and thus the distance from the true solution.

Data is presented in Table 1, x_{AM} is in the second column and the exact solution, $x(t)$, generated by Eq (2) over each time interval is presented in Figures 2 to 6. Runge Kutta solutions are also presented in these figures.

Table 2 provides the results of the inner product $y^T(t)x(t)$ and $y^T(t)\ x_{AM}$ over each time interval. Clearly $y^T(t)\ x_{AM}$ is not constant and progressively moves away from the exact solution $x(t)$ with time. This is shown in Figure 7. We have a clear measure of how x_{AM} deviates from the true solution. Thus, the inner product criteria offers a simple method for testing any linear homogeneous Lanchester model of combat for all battles.

In conclusion, we have demonstrated that the inner product criteria using the adjoint differential equation can provide combat modelers with a useful simple criteria for evaluating the performance of approximate solutions to the linear homogeneous Lanchester equation of combat.

FIGURE 1



X = EXACT LANCHESTER SOLUTION

Y = ADJOINT SOLUTION

X_{AM} = ARMY MODEL SOLUTION

$Y^T X_{AM}$ = MEASURE OF DISTANCE FROM TRUE SOLUTION X.

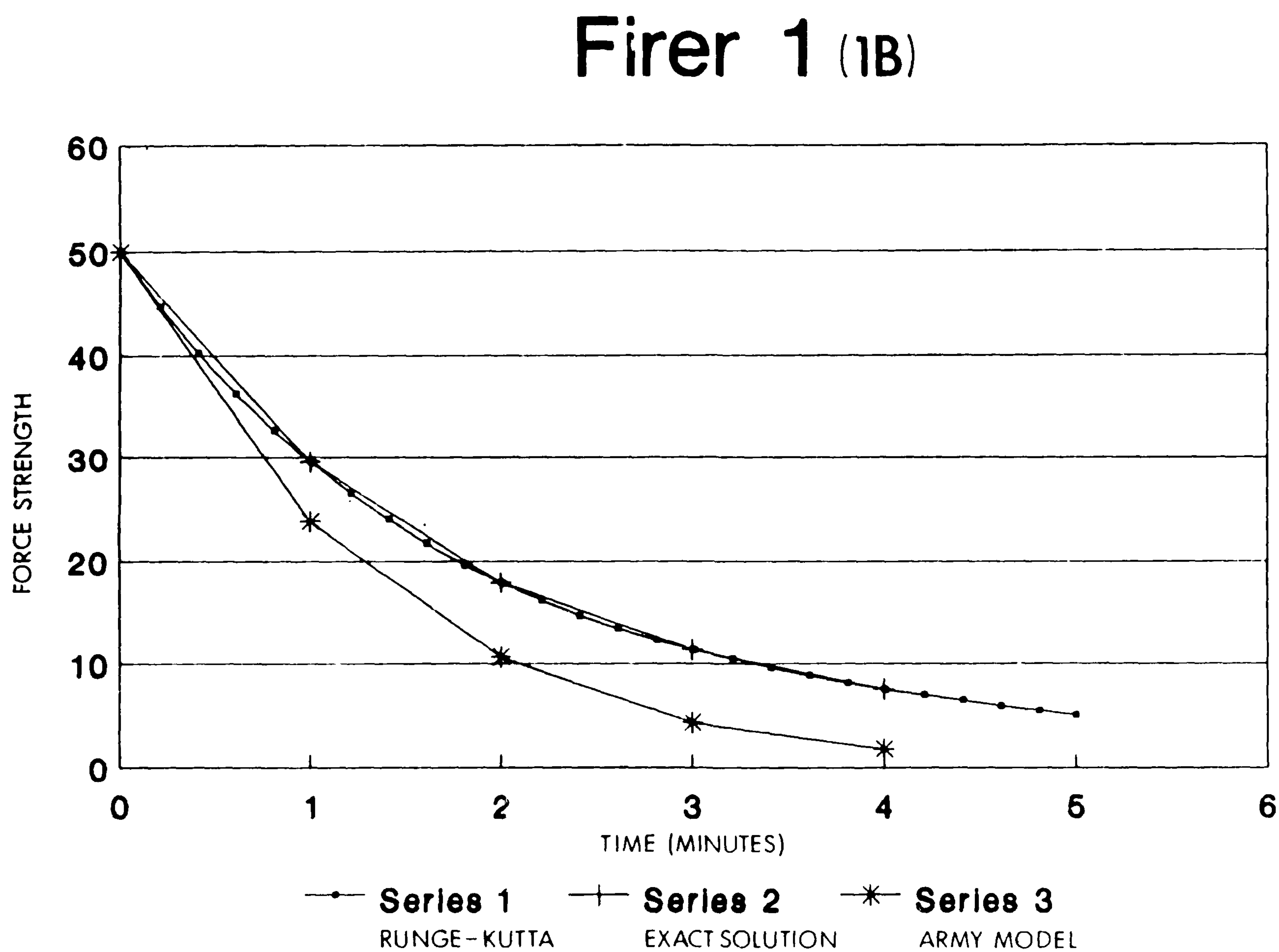
TABLE 1. DATA.

Two-on-Three Scenario
(With Suppression Off and 100% in Quadrant)

1 Blue Initial Strength, 50
2 Blue Initial Strength, 50
3 Red Initial Strength, 50
4 Red Initial Strength, 25
5 Red Initial Strength, 25

Firer	No. of Firers	Target 1	Target 2	Target 3	Target 4	Target 5
				Time = 0 minutes		
1B	50.0000	0.	0.	0.0063643	0.0000000	0.0001014
2B	50.0000	0.	0.	0.0024091	0.	0.
3R	50.0000	0.0083374	0.	0.	0.	0.
4R	25.0000	0.0008168	0.	0.	0.	0.
5R	25.0000	0.	0.	0.	0.	0.
				Time = 1 minute		
1B	23.7626	0.	0.	0.0043994	0.0000000	0.0023342
2B	50.0000	0.	0.	0.0023752	0.0000000	0.0000519
3R	23.6800	0.0083374	0.	0.	0.	0.
4R	24.9999	0.0008168	0.	0.	0.	0.
5R	24.6958	0.	0.	0.	0.	0.
				Time = 2 minutes		
1B	10.6916	0.	0.	0.0024300	0.0000816	0.0048297
2B	50.0000	0.	0.	0.0020609	0.0000124	0.0005698
3R	10.2977	0.0083343	0.0000027	0.	0.	0.
4R	24.9896	0.0008168	0.	0.	0.	0.
5R	21.2119	0.	0.	0.	0.	0.
				Time = 3 minutes		
1B	4.3175	0.	0.	0.0004420	0.0005786	0.0068515
2B	49.9983	0.	0.	0.0007079	0.0001323	0.0024820
3R	2.6462	0.0079945	0.0003013	0.	0.	0.
4R	24.9000	0.0008046	0.0000623	0.	0.	0.
5R	16.4044	0.	0.	0.	0.	0.
				Time = 4 minutes		
1B	1.8461	0.	0.	0.0000143	0.0035953	0.0034569
2B	49.8574	0.	0.	0.0001091	0.0009374	0.0025676
3R	0.4079	0.0062070	0.0018716	0.	0.	0.
4R	24.3531	0.0006816	0.0006921	0.	0.	0.
5R	7.1839	0.	0.	0.	0.	0.

Red unit removed after 4 minutes.


Firer 1 (1B)
60
50
40
30
20
10
0
FORCE STRENGTH
0
1
2
3
4
5
6
TIME (MINUTES)
Series 1
RUNGE-KUTTA
Series 2
EXACT SOLUTION
Series 3
ARMY MODEL

FIGUE 2

Firer 2 (2B)

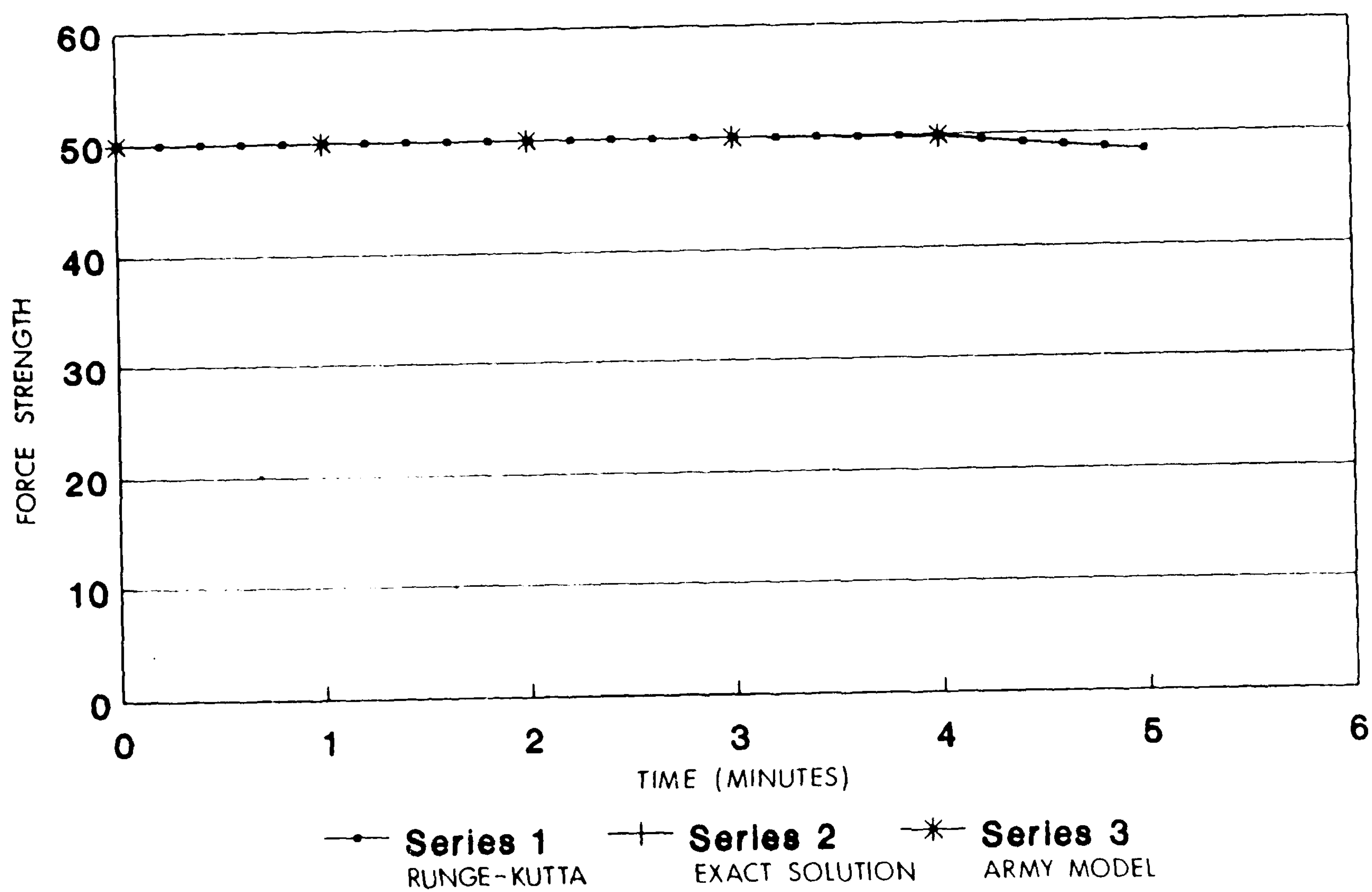
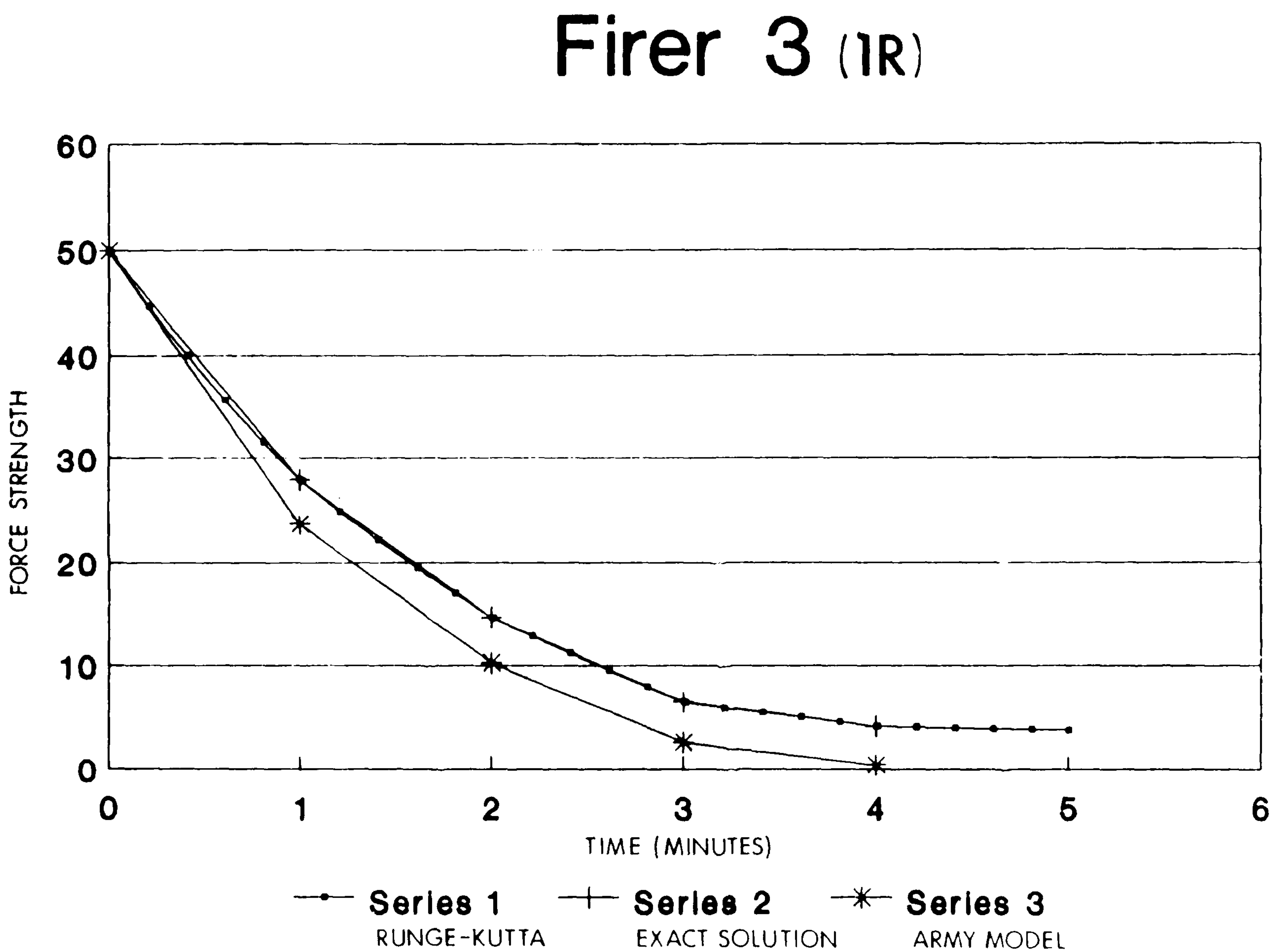


FIGURE 3



Firer 3 (1R)
60
50
40
30
20
10
0
FORCE STRENGTH
0
1
2
3
4
5
6
TIME (MINUTES)
Series 1
RUNGE-KUTTA
Series 2
EXACT SOLUTION
Series 3
ARMY MODEL

FIGURE 4

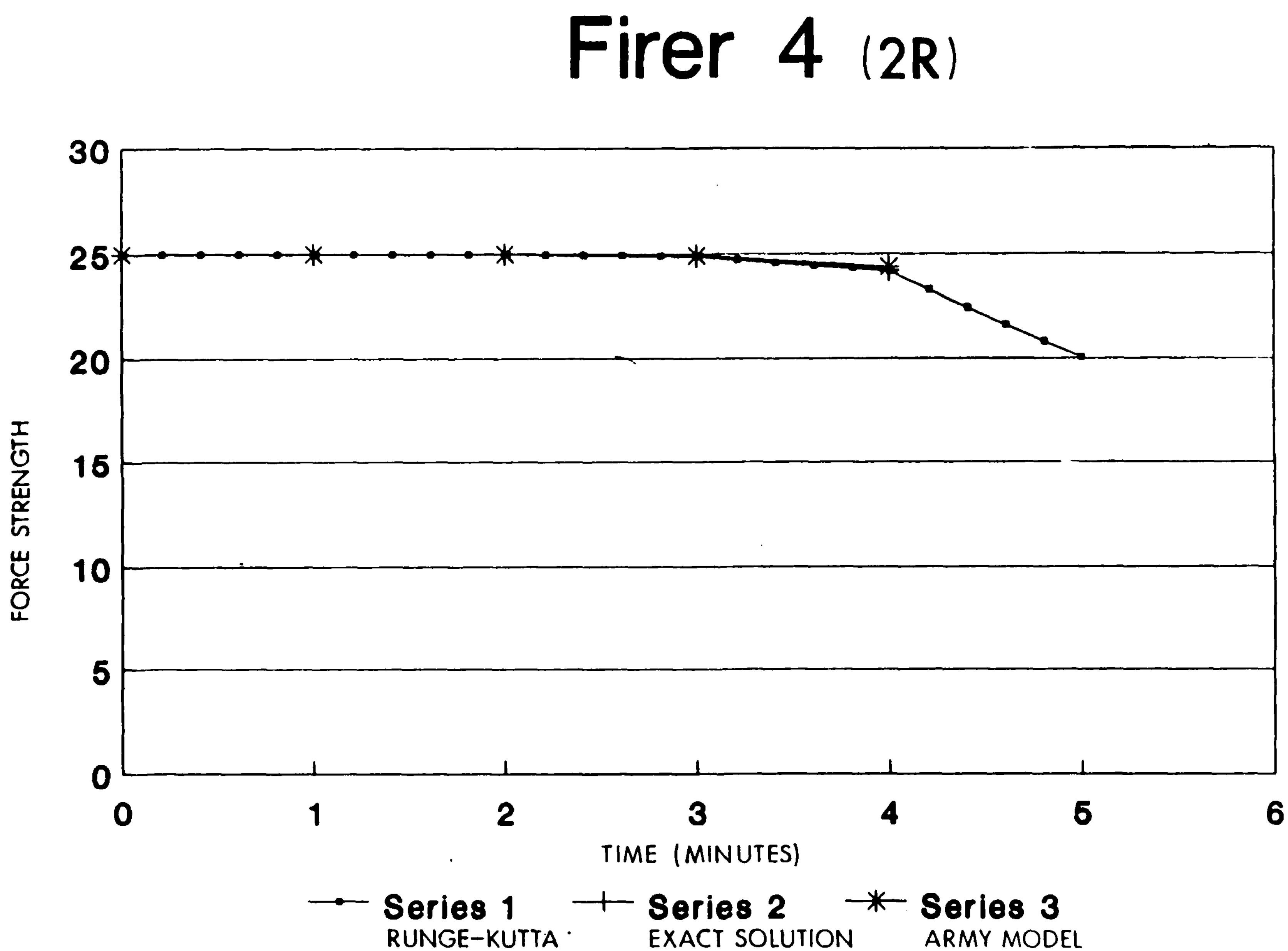
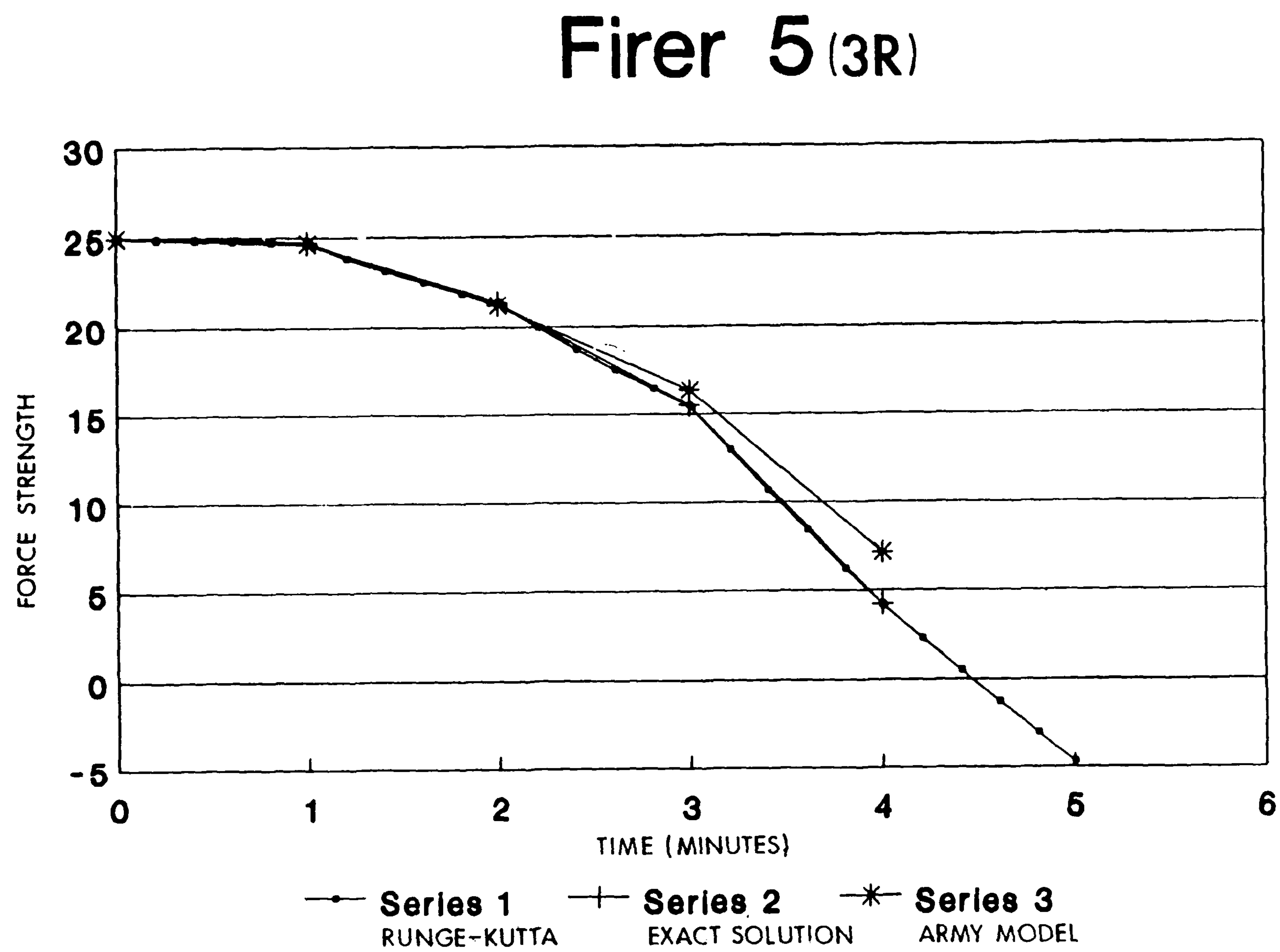

Firer 4 (2R)
30
25
20
15
10
5
0
FORCE STRENGTH
0
1
2
3
4
5
6
TIME (MINUTES)
Series 1
RUNGE-KUTTA
Series 2
EXACT SOLUTION
Series 3
ARMY MODEL

FIGURE 5



Firer 5 (3R)
30
25
20
15
10
5
0
-5
FORCE STRENGTH
0
1
2
3
4
5
6
TIME (MINUTES)
Series 1
RUNGE-KUTTA
Series 2
EXACT SOLUTION
Series 3
ARMY MODEL

FIGURE 6

TABLE 2.

$y^T(t)x(t)$

t	Initial t=0	Initial t=5
0	0	-4.16×10^{-15}
1	-2.58×10^{-14}	1.48×10^{-15}
2	-1.914×10^{-14}	-4.07×10^{-15}
3	-3.689×10^{-14}	-6.31×10^{-17}
4	-4.893×10^{-14}	2.49×10^{-16}

$y^T(t)\ x_{AM}$

t	$y^T(t)\ x_{AM}$
0	0
1	-8.6
2	-14.26
3	-17.756
4	-18.7

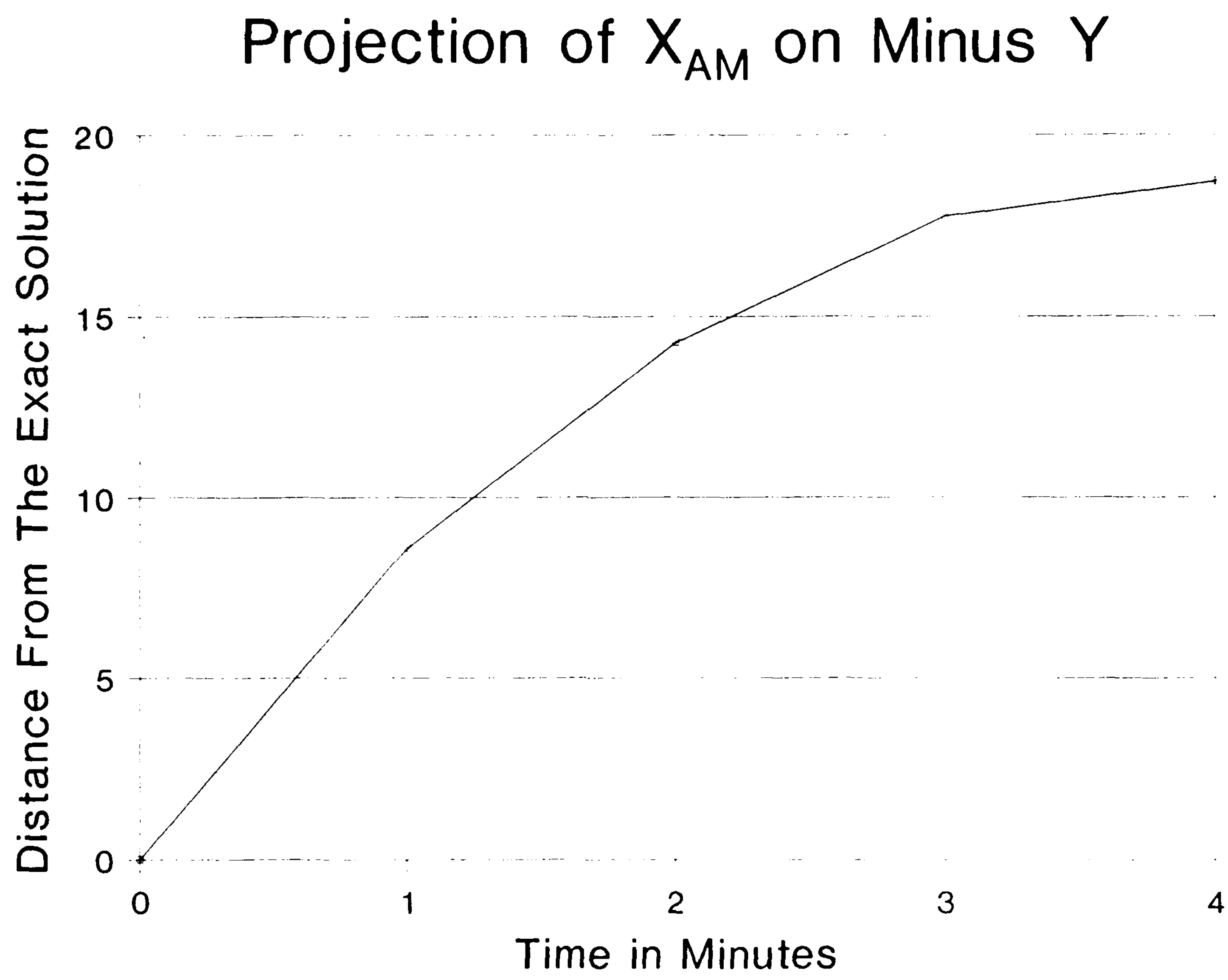

Projection of X_{AM} on Minus Y
Distance From The Exact Solution
20
15
10
5
0
0
1
2
3
4
Time in Minutes

Figure 7.

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SUBJECT: Inner Product Performance Criteria for Evaluationing Combat Models.

REASON FOR PERFORMING THIS EFFORT: Investigation of the properties of the linear homogeneous Lanchester equations which are used extensively by Army combat modelers.

MAIN OBJECTIVES OF THE EFFORT: Investigate invariant properties of liniar homogeneous Lanchester equations of combat and to compare theoretical results with those derived from Army combat models.

SCOPE OF EFFORT: Develop solutions for piecewise constant linear homogeneous Lanchester equations in state variables and introduce the role of the adjoint differential equation.

IMPACT OF THE EFFORT: Provides explicit solution, independent of step size, of the linear homogeneous Lanchester equations thus increasing speed of computation and develops a simple performance criteria for evaluating how well the approximate solutions agree with the exact solutions of the equation of combat.

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